

D(NA)-Branes

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We engineer a configuration of branes in type IIB string theory whose mechanical structure is that of a DNA molecule. We obtain it by considering a T-dual description of the quantum Hall soliton. Using a probe analysis, we investigate the dynamics of the system and show that it is stable against radial perturbations. We exercise a certain amount of restraint in discussing applications to biophysics.

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1. Introduction

String theory – at present the only known consistent theory of quantum gravity – has undergone several major changes during its lifetime in terms of the way in which physicists look at it, and what they expect from it. Originally it was viewed as a possible theory of strong interactions, although the ineluctable appearance of a massless spin-2 state in its spectrum dampened the enthusiasm for the idea. When this phenomenon was later reinterpreted as a feature rather than a bug, with the spin-2 state incarnate as a graviton rather than an anomalously light hadron, the theory garnered a reputation as the leading candidate for a theory incorporating the phenomenon of gravity into quantum mechanics. The discovery of supersymmetry, a cornucopia of compactifications to four dimensions, and quasi-realistic gauge groups and matter content in the string framework only served to solidify the hegemony of strings over the field of theoretical high-energy physics.

Much of the recent excitement in string theory has come from an (often nonperturbative) understanding of many different backgrounds of the theory and the relations between them. The protean character of quantum states under the duality group of string theory is a source of much of the continuing fascination with the theory, and of the general sense that it has many depths yet to be explored. Indeed it seems that just about any quantum theory imaginable can be obtained as a (low-energy or other) limit of the dynamics of the theory. Its shape-shifting capacity appears unlimited.

Most recently, the theory has been investigated because of the fact that one of its backgrounds seems to be able to reproduce the quantum Hall effect at low energies. In fact two logically separate dualities relating the Hall system to other theories of interest have emerged recently. The first [1] is a direct embedding of the system in string theory. The second [2] is an equivalence to a noncommutative gauge theory whose relation to string theory is less direct. This equivalence was recently sharpened in [3,4].

In this paper we describe a closely related system, obtained by considering the T-dual of (a nonabelian version of) the system in [1]. We show that the T-dual is a double helix, with fundamental string rungs connecting the two helices.

The plan of the paper is as follows. In §2, we briefly review the construction of the quantum Hall soliton, and motivate an examination of the D(NA)-brane system by considering its T-dual. In §3 we use the Dirac-Born-Infeld/Wess-Zumino action on the lighter branes to find the equilibrium radius of the helix. In §4 we describe the low-energy dynamics of the system. In §5 we conclude by discussing the limitations of our calculation and ways in which it might be improved.

2. The quantum Hall soliton and its DNA dual

We briefly review the construction [1] of a quantum Hall-like system in type IIA string theory.

Begin with a set of k sixbranes, spatially extended in the x^4, \dots, x^9 directions, located at transverse position $x_1 = x_2 = x_3 = 0$. Surrounding these sixbranes we place a set of n coincident twobranes – that is, twobranes whose location is defined by the sphere $x_1^2 + x_2^2 + x_3^2 = R_0^2$, with R_0 , the radius of the sphere, determined by the dynamics and specified later. When we say ‘surrounding’ we mean that the twobranes literally cannot be moved off to infinity without intersecting the sixbranes.

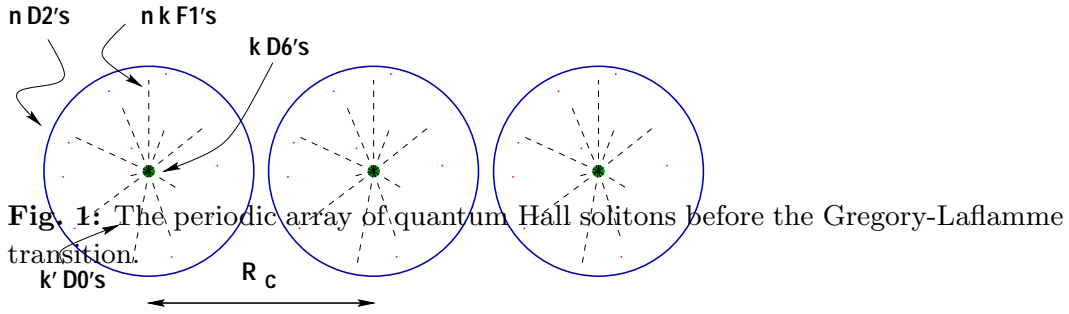
It is known [5] that for such nontrivially ‘linked’ configurations of D-branes in type II string theories there is an effect that causes fundamental strings to connect the two linked objects. Referred to as the ‘Hanany-Witten’ effect after its discoverers, this feature of brane dynamics can be understood from many different points of view, *c.f. e.g.* [6].

In order to stabilize the configuration one can dissolve n zerobranes in the set of twobranes. There is a repulsive force between zerobranes and sixbranes at long distances, and so the system seeks its equilibrium radius, calculated in [1] to be $R_0 = (\pi k'/n)^{2/3} l_s/2$.

The authors of [1] argued that the dynamics of this system were closely related to those of the quantum Hall effect, in that the string endpoints appeared in the gauge theory dynamics as charges in the fundamental representation (in [1] only a single twobrane was considered, and so the string endpoints were simply electric charges) and the dissolved zerobranes were units of magnetic flux. For a single twobrane the ‘filling fraction’ of the system was simply k/k' , the ratio of the density of electrons to the density of flux.

Now suppose one compactifies the system along the x_3 direction with radius R_c . For $R_c \gg R_0$ we can think of the periodically identified configuration as an infinite array of twobrane spheres surrounding an array of sixbranes as in fig. 1¹. We expect, however, that as we reduce the compactification radius until $R_c \sim R_0$, the system will develop an instability (closely related to the Gregory-Laflamme instability [7]) towards a merger of the twobranes into a cylinder, periodically identified in the compact direction.

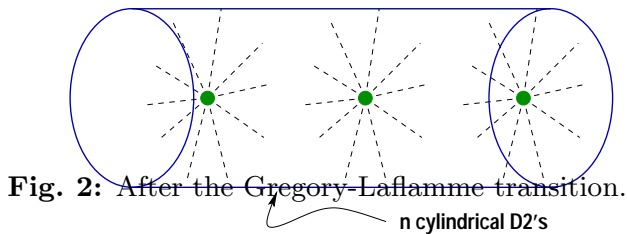
¹ Strictly speaking as soon as one compactifies transverse to the sixbranes one runs into trouble because the fields generated by the branes grow logarithmically in the remaining transverse directions and there is a conical deficit at infinity. For now we assume we can regulate this problem, but we will discuss the point in more detail further on.



Now we perform T -duality along the x_3 direction. Under this duality:

- The k' zerobranes become $D1$ -branes, extended in the x^3 direction.
- The k sixbranes become $D7$ -branes, extended in the x_3, x_4, \dots, x_9 directions.
- The n cylindrical twobranes become circular onebranes in the x_1, x_2 plane.
- The nk fundamental strings stay as they are; now they are stretched between the sevenbranes and the circular D-strings surrounding them.

In fact this description is not very accurate; it is merely a cartoon which illustrates the way in which the charges transform under T -duality. The configuration shown in fig. 3 would more accurately describe the T -dual of the unstable system in which the zerobranes are present but not dissolved in the twobranes. The correct, (meta)stable configuration in the original picture is the one in which the zerobranes and twobranes form a bound state in which the zerobranes give up almost all their rest energy. The correct T -dual picture is one in which the circular onebranes 'bind' to the straight onebranes by forming a coil – the preference of the system for the bound state is a consequence of the Pythagorean theorem.



Note that the picture above is most accurate if the ratio n/k' is large, a limit opposite that of [1], in which $n = 1$. For $n/k' \gg 1$ (so that the number of coils is large) and $R_c/\alpha' \ll 1$ (so that the T-dual radius is large) we really can consider an infinitely extended D(NA)-brane. Specifically, in order to keep fixed the number of coils per unit length of a bundle of k' D1-branes, we need to take a large- n limit of the nonabelian quantum Hall soliton:

$$n \rightarrow \infty$$

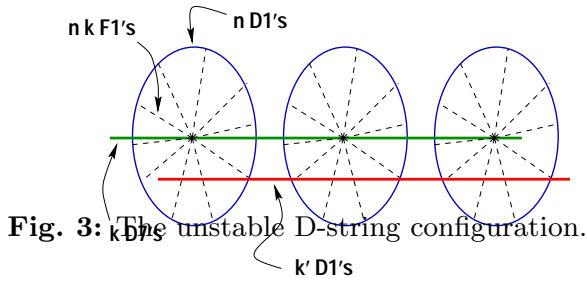
$$R_c \rightarrow 0$$

holding fixed the natural quantities in the T -dual picture:

$$\begin{aligned} k' &= \text{number of onebranes in a bunch} && \text{(fixed)} \\ \frac{nR_c}{k'\alpha'} &= \text{number of coils per unit length} && \text{(fixed)} \\ k &= \text{number of sevenbranes in a bunch} && \text{(fixed)} \end{aligned} \tag{2.1}$$

We also keep fixed the coupling in the resulting T-dual theory (this means we have to scale the original coupling). Note that the number of fundamental strings per coil is

$$\frac{\text{number of F-strings per } R_c}{\text{number of coils per } R_c} = \frac{nk}{n/k'} = kk'.$$

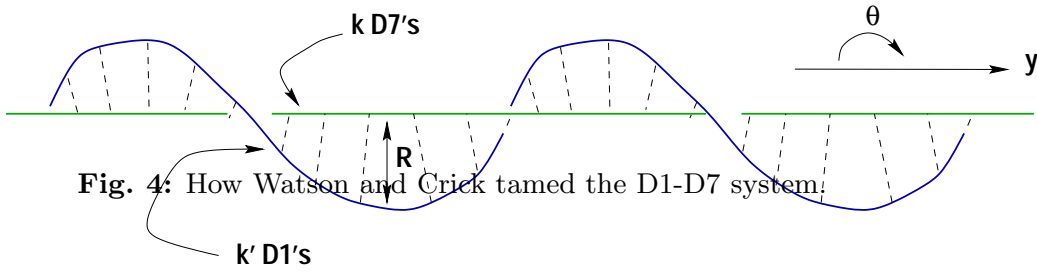


Consider for a moment taking $k' = 0$. In that case, the D-strings would not coil in the y -direction at all, and would close onto themselves. Recall that when a fundamental string ends on a D-string, the D-string carries away the F-string charge in one direction (as worldline flux).

In order to have charge conservation when the D-string closes, the monodromy action of the D7-branes on the (p, q) -string charges must cancel off this accumulated F-string charge. If we start off with a (p, q) string, and go around the k D7 branes which emit nk F-string spokes, we must have (in the notation of [6])

$$\begin{pmatrix} p \\ q \end{pmatrix} = M_{[k,0]} \begin{pmatrix} p - nk \\ q \end{pmatrix} = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} p - nk \\ q \end{pmatrix} = \begin{pmatrix} p - nk + kq \\ q \end{pmatrix},$$

where $M_{[p',q']} = \begin{pmatrix} 1 - p'q' & p'^2 \\ -q'^2 & 1 + p'q' \end{pmatrix}$ is the monodromy experienced by the charge lattice of (p, q) strings in traversing the branch cut of a (p', q') 7-brane [6]. Therefore we see that we must have $q = n$ units of D-string charge, and that the number p of units of F-string charge is arbitrary. Once we take k' nonzero, the string moves in the y direction as it coils and this condition illuminates the Hanany-Witten effect from the F-theory point of view.



The Double Helix

Finally, consider what will happen if we compactify the six dimensions x_4, \dots, x_9 on a T_6 of total volume V_6 (whose individual dimensions will not figure in the discussion), giving the wrapped sevenbranes a finite tension. This will cause them to coil up in response to the pull of the stretched fundamental strings, and the resulting object will be a double helix. The tension of the effective one-dimensional object in $3 + 1$ dimensions will be $V_6/(\alpha'^3)$ times that of a onebrane; we will let V_6 be large enough that we can treat the bundle of sevenbranes as a heavy, fixed background in which the onebranes move as probes.

Before analyzing the stability of the system, there is one final subtlety to be dodged. When one considers an infinitely extended one-dimensional object in three spatial dimensions, one must deal somehow with the fact that objects of spatial codimension two have rather dramatic behavior when they couple to massless fields, particularly to the metric. The sevenbranes and onebranes are sources for the RR zero-form and two-form potentials,

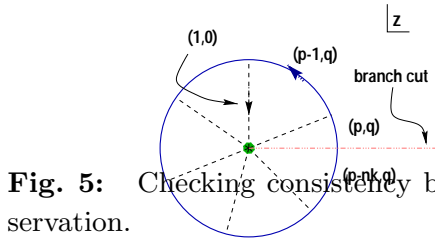


Fig. 5: Checking consistency between sevenbrane monodromy and charge conservation.

respectively, which grow logarithmically rather than falling off at large distance from the source. Also the coupling to the metric creates a conical deficit at infinity. Furthermore if one tries to add too many sevenbranes and onebranes, one will actually drive the deficit beyond 360° . Finally both the onebranes and sevenbranes source the dilaton and other moduli, which grow logarithmically away from the branes, changing the way in which one defines the adjustable coupling for this background.

While there may be ways to regulate the long-distance fields of an infinite string consistently, we take a strictly pragmatic approach in this paper and instead of an infinite D(NA)-string, we consider an object with a different “tertiary structure”.

Wind the helix around in a large loop with a radius far larger than the string scale, the radius of the helix, or the scale on which the helix is coiled.

Obviously, if we make it big enough, the timescale for its collapse will be much longer than those relevant to the “secondary structure”. Among other simplifying features, this regulated configuration has unambiguous expectation values of the dilaton and other moduli at infinity, although the dynamics on the onebranes will decouple from the asymptotic dilaton.

3. Probe analysis and low-energy dynamics

The complexified dilaton is

$$\tau(z) = \chi + ie^{-\Phi}.$$

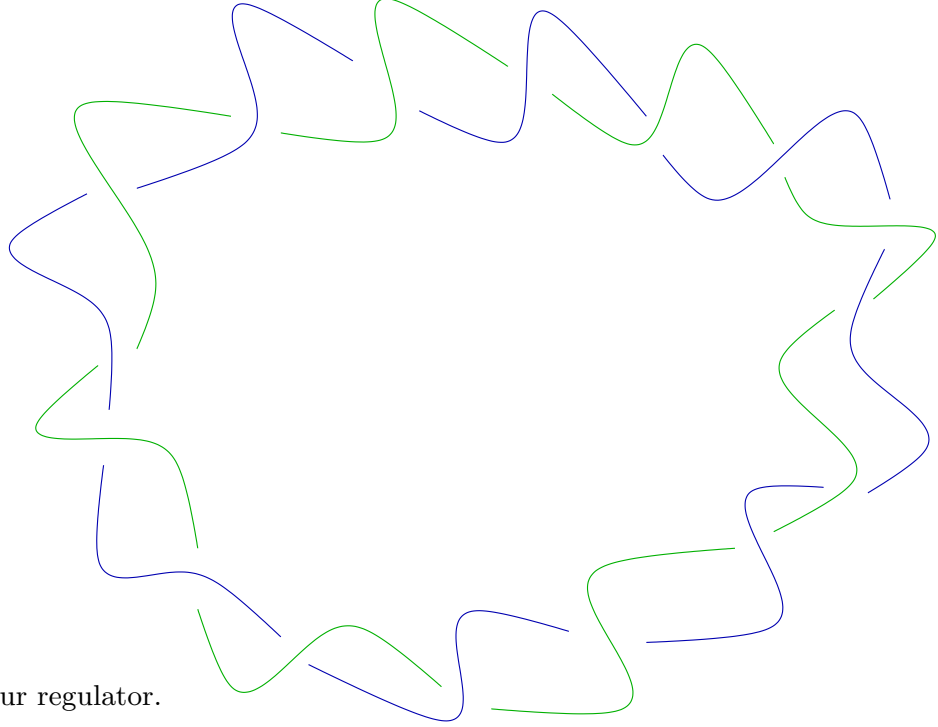


Fig. 6: Our regulator.

The Einstein-frame metric for sevenbranes with worldvolume along $t, y, x_4, \dots x_9$ is [8]

$$ds_{\text{ein}}^2 = -dt^2 + dy^2 + \sum_{i=4}^9 dx_i^2 + \Omega^2 dz d\bar{z},$$

where Ω is a function of z defined below. The string-frame metric is $ds^2 = \frac{1}{\sqrt{\text{Im}\tau}} ds_{\text{ein}}^2$. We work in units where $\alpha' = 1$. Let $z = re^{i\theta}$.

We take the near-brane solution for a stack of k D7-branes, for which the complexified dilaton takes the form

$$\tau(z) = j^{-1}(-bz^k) \simeq \frac{k}{2\pi i} \ln bz.$$

This approximation is valid at weak coupling, *i.e.* when $\tau_2 \equiv \text{Im}\tau$ is big. b is a parameter of the solution which plays the role of a dilaton modulus. The metric function Ω takes the form

$$\Omega^2 = \tau_2 |\eta(\tau)|^2 \prod_i (z - z_i)^{-\frac{1}{12}}|^2 \simeq \tau_2 |(bz)^{\frac{2k}{24}} z^{-\frac{k}{12}}|^2 \simeq -\frac{k}{2\pi} b^{\frac{k}{6}} \ln(bR) \equiv -\frac{k}{2\pi\gamma^2} \ln(bR),$$

where η is the Dedekind eta function.

Parametrize the worldvolume of the helix by a spatial coordinate σ and choose the embedding

$$y = \alpha\sigma, \quad z = Re^{i\sigma},$$

so that $\theta = \sigma \bmod 2\pi$. α is the inverse number of coils per unit length as defined by the scaling limit (2.1).

Then, ignoring kinetic terms because at the moment we are just interested in finding the potential for R , the induced $E = G + F$ (using the string-frame metric) is

$$\begin{aligned} E &= (G + F)_{\alpha\beta} d\sigma_\alpha d\sigma_\beta = (G^{\mu\nu} \partial_\alpha X_\mu \partial_\beta X^\nu + F_{\alpha\beta}) d\sigma^\alpha d\sigma^\beta \\ &= (dt, d\sigma) \begin{pmatrix} -\frac{1}{\sqrt{\tau_2}} & \xi \\ -\xi & \frac{1}{\sqrt{\tau_2}}(R^2 \Omega^2 + \alpha^2) \end{pmatrix} \begin{pmatrix} dt \\ d\sigma \end{pmatrix}, \end{aligned}$$

where $\xi = F_{0\sigma}$ is the electric field along the helix. So we have

$$-\det E = e^\Phi (\Omega^2 R^2 + \alpha^2) - \xi^2.$$

Let us discuss the form of ξ . The sources for the field strength ξ are the endpoints of the fundamental strings on the D-string, and the background axion gradient:

$$\mathcal{L}_\xi \sim \frac{1}{g_{YM}^2} \xi^2 - A_0 \left(\frac{1}{k'} \sum_i \delta(\text{string end}_i) - \partial_1 \chi \right),$$

where $g_{YM}^2 = e^\Phi$ and $\xi d\sigma dt = dA$. The factor of $\frac{1}{k'}$ multiplying the string source arises because a single string-end ending on a clump of D-strings sources the trace of ξ with unit strength. We find that the solution should be

$$\begin{aligned} \xi &\sim e^\Phi \left(\frac{k\sigma}{2\pi} - \frac{2\pi}{k'} \sum_i \Theta(\text{string end}_i) \right) - \xi_0 \\ &= \frac{-2\pi}{\ln bR} \left(\sigma - \frac{2\pi}{kk'} \sum_i \Theta(\text{string end}_i) \right) - \xi_0. \end{aligned}$$

ξ_0 is a background electric field determined by the worldline theta-angle, which is in turn related to the bulk RR axion. As we will explain below, the worldline theta-angle becomes dynamical and chooses ξ_0 to make the average electric field vanish.

The electric field on the D-strings has the form of a sawtooth which reaches its maximum when it reaches a string end which then discharges it. However, in the following, we take a average field approximation, where we replace ξ by its spatial average value. This approximation is justified by the fact that the deviation of ξ from its average value is of order $\frac{1}{kk'}$ which we take to be small.

Plugging into the Dirac-Born-Infeld plus Wess-Zumino probe action,

$$S = \int dt d\sigma \mathcal{L} = k' \int dt d\sigma \left(e^{-\Phi} \sqrt{-\det E} + e^{-(F+B)} \sum_p C^{(RR)} \right),$$

(the k' is out front because there are k' D-strings) we find

$$\mathcal{L} = k' \sqrt{\tau_2 \left(\left(\frac{R^2 \tau_2}{\gamma^2} \right) + \alpha^2 - \tau_2 \xi^2 \right)} + \frac{k k' \sigma}{2\pi} \xi. \quad (3.1)$$

We must also include the force on the D-string from the tension of the attached fundamental strings. These contribute an energy proportional to their length in the string frame metric, which is in turn equal to their coordinate length, over γ . Averaging over σ , we find that they contribute a linear potential,

$$V_{\text{strings}} = \frac{k' k}{2\pi \alpha' \gamma} R.$$

In the average field approximation, a convenient way to write the potential for $x = bR$ is

$$V = \frac{k k'}{2\pi \gamma b} \left(x + \sqrt{x^2 \ln^2 x - c \ln x} \right) \quad (3.2)$$

where $c \equiv 2\pi \alpha'^2 b^2 \gamma^2 / k$, a dimensionless parameter. For small but nonzero c , this function has a minimum at x_E defined by the transcendental equation

$$c = 2x_E^2 \left(\ln^2 x_E + \sqrt{2} |\ln x_E|^{3/2} \right).$$

There is a critical value of $c \simeq 0.6689$ above which there is no minimum, and the potential just slopes off toward infinity.

Issues raised by this calculation

1. Obviously we have only considered the dynamics of a single mode of the helix, albeit the most obvious candidate for an instability. In the next section, we consider some other modes.
2. The sevenbrane geometry which we are probing is singular if $k \neq 24$. Further, there is a range of r 's ($r > 1/b$) for which our near-brane approximation breaks down and the dilaton, Φ , is apparently imaginary. However, the minimum we found above lies within the region where our approximation makes sense.

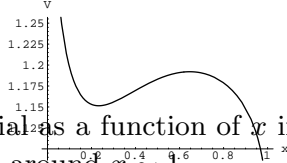


Fig. 7: The potential as a function of x in units of $\frac{kk'}{2\pi\gamma b}$ for $c = 0.5$. The gravity solution goes stupid around $x \sim 1$.

3. The DBI analysis is only valid if the brane worldvolume is weakly curved, and its field strengths are slowly varying; this is the case for our probe, except for the step-function discontinuities due to fundamental string sources.
4. The value of the dilaton at the D-strings,

$$e^{\Phi(R_E)} = \frac{2\pi}{k|\ln x_E|}$$

can be made parametrically small by taking k large and α^2 large fixing R_E, c, b .

5. If we take c smoothly to zero, the minimum we found above moves closer to the sevenbranes. In this limit, the 1-7 strings become light, but the coupling to the 7-7 strings can be kept small by making V_6 large. Since $c \propto \alpha^2$, this is consistent with the T-dual fact that in the absence of zero-branes, the quantum Hall soliton collapses.
6. Observe in fig. 7 that our potential has a maximum, and slopes downward far from the sevenbranes. When c is small, this maximum is inside the region of validity of our approximation. This signals that our equilibrium is perhaps only metastable, as is the quantum Hall soliton.

4. Low-energy theory on the strings

What can we say about the effective dynamics on the worldline of the helix? The low-energy effective excitations are as follows:

- There are six compact scalars from the transverse T^6 . The T^6 is large and the sevenbranes are distant, and we assume in what follows that these modes decouple. There is also a seventh Goldstone mode, corresponding to translation in the axial direction, which is likely to decouple as well.

- We found that there is a (stable) equilibrium value for the radion field, R , that measures the coordinate distance between the onebranes and sevenbranes; the radion is massive.

- There is a charge displacement wave mode, D , which is massive. In the presence of a neutralizing background charge (provided by the axion gradient), the charged string endpoints are bound to their equilibrium positions with a linear restoring force.

- There is a mode that rotates the entire onebrane around the sevenbranes, leaving the F -strings at fixed axial position. This 'turnon'² field T is not an independent mode; it can be compensated by a combined charge displacement and axial shift. This corresponds with the fact that in the presence of a background axion that winds k times around the sevenbranes, the turnon is in effect a dynamical theta angle in $1 + 1$ dimensions; giving T a vev produces a vacuum energy proportional to T^2 which matches the energy from the corresponding charge displacement.

- There are modes corresponding to fragmentation of the onebrane into constituent onebranes. There are also fragmentation modes along the T^6 directions. Understanding the fragmentation modes would require a more careful treatment of the nonabelian dynamics than we attempt here. In the probe analysis, then, we will set $k' \equiv 1$. However abelian and nonabelian coulomb forces are quite similar in $1 + 1$ dimensions, and so we believe the qualitative picture may be similar when $k' \neq 1$.

- There is no independent 'unwindon' – that is, the mode which uncoils the D-strings is essentially a linearly rising mode of the turnon field.

4.1. The full potential

In this subsection, we perform a refinement of the calculation of §3, for the case $k' = 1$.

Let us fix $y \equiv \alpha\sigma$, $X^0 \equiv \tau$ identically as a helical analog of static gauge for the reparametrization invariance of the DBI action. In order to find the roton and turnon potential, we write

$$z \equiv R(\sigma)\exp\{i(\sigma + T(\sigma))\} \quad (4.1)$$

² Since the word 'roton' already has a standard usage in condensed matter physics, we were left with little choice in the matter.

First we set the charge density to its equilibrium value, and find a potential for R and T alone. We apply the approximation in which the fundamental string endpoints are continuously and uniformly distributed so as to cancel the background charge density coming from the axion gradient. The electric field on the onebrane worldvolume is constant, and equal to ξ . The energy is at a minimum when ξ vanishes. In this system the θ angle is dynamical, so the electric field can relax to zero by changing the value of the turnon.

Substituting this parametrization into the probe action we expand to zeroth order in derivatives, for the full Lagrangian:

$$\mathcal{L}(R, T) = e^{-\Phi(R)} \sqrt{\alpha^2 + R^2 \Omega^2(R)} - \xi^2 + T\xi - \frac{k}{2\pi\gamma} R. \quad (4.2)$$

Notice that the turnon field only appears in the WZ term, coupling as a Peccei-Quinn axion. The background electric field is determined by the effective theta angle:

$$\xi = \frac{\tilde{\tau}_1}{\sqrt{\tilde{\tau}_2} |\tilde{\tau}(R, T)|} \cdot \sqrt{\alpha^2 + R^2 \Omega^2(R)}$$

where $\tilde{\tau} \equiv T + i \cdot e^{-\Phi(R)}$ is the effective gauge coupling. Note that ξ has an interpretation as a density of dissolved Wick-rotated D(-1)-branes. Integrating out ξ in the manner above is analogous to the “dilute instanton gas” approximation in four-dimensional axion physics.

Plugging this back into the Lagrangian we obtain

$$V = -e^{\Phi(R)/2} \sqrt{\alpha^2 + R^2 \Omega^2(R)} \cdot |\tilde{\tau}(R, T)| + \frac{k}{2\pi\gamma} R. \quad (4.3)$$

Having done this calculation, it is easy to take account of the motions of the charged string endpoints. A charge distribution couples to the gauge field as

$$L \rightarrow L + \rho(\sigma) A_0(\sigma). \quad (4.4)$$

However for small motions of the charges from their equilibrium positions, the charge density is given by the *inhomogeneity* of the charge displacement field $D(\sigma)$; that is $\rho(\sigma) = D'(\sigma)$, so integrating by parts we find the interaction term

$$-D(\sigma)\xi. \quad (4.5)$$

So T and D enter the action only through their difference. To find the full potential for all the fields, including the charge displacement, simply substitute $T - D$ for T in the

expression (4.3). This expresses the fact that any uniform displacement of the charges from their equilibrium positions can be compensated by a rotation of the helix.

In the end we find a $(1 + 1)$ -dimensional system of three coupled fields: the charge density of string-ends, the turnon field, and the radion. In the absence of a charge-clumping instability, we expect that the radion field decouples. The resulting system seems to form a Wigner crystal. It will be interesting to learn more about the D(NA)-brane system, particularly when $k' \neq 1$.

5. Conclusions

We have shown that the quantum Hall soliton has a certain limit in which it is naturally viewed via T-duality as a molecule of DNA. Though there is still much we do not understand about the D(NA)-brane system, the dynamics are those of point charges in a neutralizing background. In addition to the Goldstone modes, the theory on the strand contains a worldvolume axion, the turnon, and a charge displacement field, one combination of which is massless.

Our computation is incomplete in the following ways:

- In our low-energy analysis, we have merely determined which modes have nonzero mass. In order to compute physical masses of fields, one would need to compute their kinetic terms.
- Where uncertain, we have given the benefit of the doubt to approximations and assumptions which emphasize the possible similarity of our system to that of [1]. In particular, we have treated the string endpoints on the sevenbranes as if they could be effectively decoupled; we leave open the problem of treating them more realistically.
- We have not attempted to understand the nonabelian worldvolume dynamics when $k' > 1$, particularly whether or not there may be a “genes’ instability” to fragmentation of the clump.
- A better analysis of the effective dynamics should take into account that the lowest modes of the stretched fundamental strings are fermionic [1].
- It would be interesting to go beyond the average-field approximation for the charge distribution to see if an electric analog of the structure of [2] emerges.

It would be interesting to learn more about this system.

6. What we have to say about biophysics

Consider the case of two sevenbranes and two D-strings. Let the fundamental index of the D-string gauge group run over “purine” and “pyrimidine” . . .

Acknowledgements

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